Engineering Notes

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An Approach to Penetration Cost Effectiveness Evaluation

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Introduction

THE method described is suggested as one tool for making first-order approximations in the evaluation of systems for improving aircraft survivability in a hostile environment. It is an attempt to quantify for engineering purposes a number of factors that are already known intuitively. The analysis first presents a criterion for comparing the cost of two or more alternatives, each of which must accomplish the same function—penetration. This is an equal-effectiveness criterion. Second, the analysis contains a discussion of the variables affecting the cost effectiveness of survivability and the sensitivity of the criterion to the major parameters among those variables.

The mission objective is defined as requiring a given number of airplanes to pass over a target, and the measure of cost effectiveness chosen is the total cost of one or more alternatives to accomplish this objective. The objective may be achieved either by increasing the number of airplanes in the overflight according to expected losses, by providing passive defense measures or defense suppression aids to increase the probability of survival of the aircraft, or both. The factors in the total cost include the direct cost of aircraft lost, the direct cost of the defensive subsystems added to reduce losses, and the airplane or mission performance penalty incurred by carrying defensive subsystems. The direct costs of any equipment include unit investment costs with prorated R&D and the costs associated with operation and maintenance for the life of the equipment.

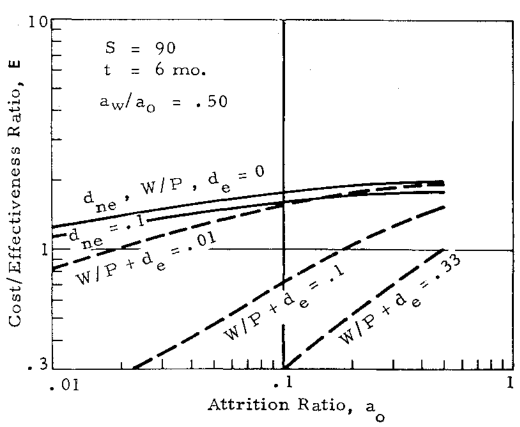

Fig. 1 Cost/effectiveness variation with attrition rate: comparison of costs and weight penalties, short medium conflict.

Presented as Paper 66-782 at the AIAA Military Aircraft Systems Meeting, Dallas, Texas, October 18–19, 1966; submitted November 2, 1966; revision received May 31, 1967. This paper was a result of research funded by The Bendix Corporation. [1.07, 2.04, 9.02]

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Development of Equations

As suggested previously, penetration aid cost effectiveness can be expressed as the ratio of the total cost of the undefended aircraft to perform a given penetration, C_0, to the cost of defense-equipped aircraft required to perform the same task, C_w. Thus a payoff will accrue when this cost effectiveness ratio is greater than unity. The cost effectiveness ratio E can then be expressed as

$$E = C_0/C_w \tag{1}$$

In Eq. (1), C_0 is the sum of all the direct costs of a basic airplane surviving to the target area without defensive subsystems; or

$$C_0 = C_n + C_M + C_l \tag{2}$$

where

C_n = the total cost of all airplanes surviving
C_M = operation and maintenance cost of those airplanes
C_l = cost of airplanes lost penetrating to the target

These terms can be derived in the following manner:

$$C_n = A_nC$$

where

A_n = the number of airplanes surviving
C = the unit investment cost plus prorated RDT&E costs

The nominal lifetime T of an aircraft is assumed to be five years; and the operation and maintenance (O&M) costs are approximately four times the RDT&E plus production costs. Therefore, the O&M costs for the surviving airplanes $C_M = A_nC(4t/T)$. The cost of lost airplanes,

$$C_l = (A - A_n)CSB$$

where

A = the number of airplanes starting the mission
S = the number of sorties during the theater operational period

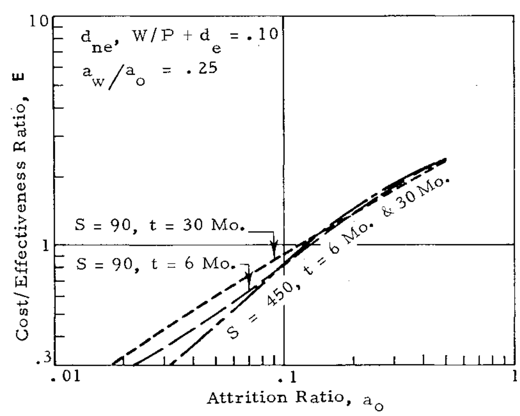

Fig. 2 Cost/effectiveness variation with attrition rate: effect of sortie rate & theater duration.

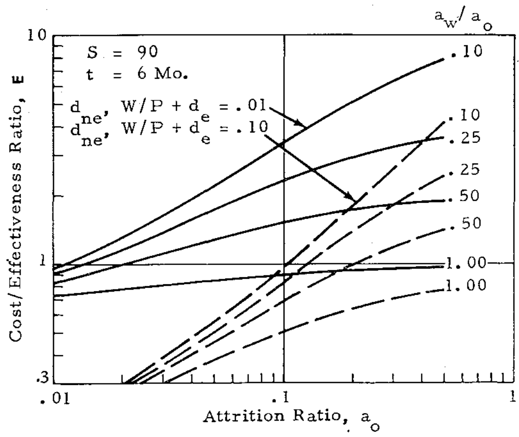

Fig. 3 Cost/effectiveness variation with attrition rate: various improvement ratios, costs, & weights, short medium conflict.

B = a loss logistics adjustment factor to account for the cost of replacing an airplane in the theater

By substituting C_n, C_M, and C_l and arranging, Eq. (2) can be written as

$$C_0 = A_nC(1 + 4t/T) + SBC(A - A_n) \tag{3}$$

which represents the cost of aircraft surviving plus the cost of aircraft lost. The attrition ratio of aircraft without penetration aid defenses is $a_0 = (A - A_n)/A$, and therefore $(A - A_n) = A_na_0/(1 - a_0)$. Substituting for $(A - A_n)$, Eq. (3) can then be rewritten

$$C_0 = A_nC[1 + 4t/T + SBa_0/(1 - a_0)] \tag{4}$$

The term C_w in Eq. (1) is the sum of all the direct costs of the basic aircraft modified by the addition of defensive subsystems in the following manner. First, if additional defensive subsystems are effective, the mission costs will be reduced in part directly as the ratio of the losses. Second, the mission costs will be increased directly by the added cost of penetration aids. Third, the cost of a mission will be increased indirectly because the performance of each airplane carrying defensive subsystems will be degraded, mainly by a reduction in payload; or on a total mission basis if special defense-suppression aircraft were used, the cost of additional aircraft can be represented by an equivalent payload penalty.

The cost of aircraft with penetration aids, C_w, is then equal to the cost of airplanes surviving including O&M costs, plus the cost of airplanes lost, plus the cost of defensive subsystems surviving including O&M costs, plus the cost of defensive subsystems lost. This may be written

$$C_w = C_n + C_M + C_l + C_{d_n} + C_{dl} \tag{5}$$

Taking the same approach used in the derivation of Eq. (3),

$$C_n = A_nC \qquad C_M = A_nC(4t/T)$$

$$C_l = (A - A_n)CSBa_w/a_0$$

$$C_{d_n} = [A_nCd_{ne} + A_nCd_{ne}(4t/T)] + [A_nCSd_e + A_nCSW/P]$$

$$C_{dl} = (A - A_n)CSB(a_w/a_0)(d_{ne} + d_e + W/P)$$

where the new terms are

a_w = the attrition ratio of aircraft with defensive subsystems

d_e = the fractional cost of expendible defensive subsystem elements (missiles, bullets, etc.) relative to airplane unit cost C

d_{ne} = the relative cost of nonexpendible defensive subsystem elements

W/P = the fractional payload penalty incurred by carrying penetration aids

By substitution into Eq. (5) and arranging we can write

$$C_w = A_nC(1 + 4t/T) + SBC(A - A_n)a_w/a_0 + A_nC[S(d_e + W/P) + d_{ne}(1 + 4t/T)] + SBC(d_e + W/P + d_{ne})(A - A_n)a_w/a_0 \tag{6}$$

Substituting for C_0 and C_w in Eq. (1) gives

$$E = \frac{1 + 4t/T + SBa_0/(1 - a_0)}{1 + 4t/T(1 + d_{ne}) + S[d + W/P + Ba_0/(1 - a_0)a_w/a_0(1 + d + W/P + d_{ne})]} \tag{7}$$

This equation then defines a restricted form of cost effectiveness criterion, specialized for use in establishing the relative value of two penetration alternates.

Sensitivity of Criteria to Parameters

Typical results are presented in Figs. 1–3 as parametric plots of cost effectiveness ratio E vs undefended aircraft attrition ratio a_0. The term a_0 is an indicator of the quality and quantity of antiair defenses and the kind of missions flown; higher values indicating a more severe environment. The results are plotted so that effects of those variables to which E is most sensitive can be readily seen.

It is the nature of this kind of analysis that the answers appear to be reasonable. As expected, the payoff for airborne defensive systems varies significantly with a_0. From Fig. 1, it is noted that payload penalties W/P and cost of expendibles d_e are stronger factors than cost of nonexpendables. From Fig. 2, it appears that the time in the theater t and the number of sorties flown S are not particularly important to this kind of comparison probably because the loss and O&M costs are treated about the same for the penetration aids as for the basic airplanes. On the other hand, loss improvement ratio a_w/a_0 is a very powerful parameter. This factor defines the value of the penetration aids in relation to the antiair defenses.

If we assume that a sophisticated airborne defense system may require about 10% of the total mission payload and cost as much as 10% of the basic aircraft, two other observations can be made: first, the level of war and aircraft utilization must be such that a_0 is greater than about 0.10 before complex and expensive penetration aids will be worth considering; and second, it is important to achieve an improvement in survivability of about 2 to 1 ($a_w/a_0 = 0.5$), in order

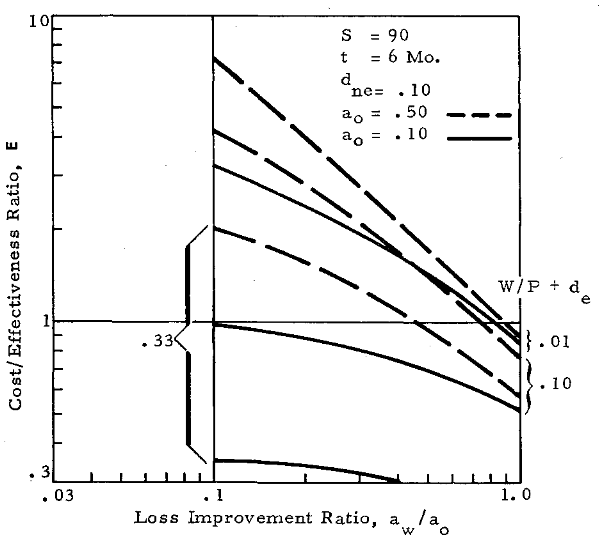

Fig. 4 Effect of loss improvement ratio on cost effectiveness: short medium conflict, comparison at moderate & high attrition.

to realize a payoff even in rather severe hostile environments. Further, it appears that defensive systems with weights above about 20% of the mission payload will pay off only in the most severe environments ($a_0 > 0.2$) and then only if costs are moderate. The cross plots in Fig. 4 tend to bear out the foregoing conclusions in a different manner. For the severe operational environment case ($a_0 = 0.50$) it appears that a penetration aid offering an improvement better than 2:1 in survivability will pay off if its costs are less than about 10% of the combined mission equipment and total weights are less then 10% of the combined mission payload. Yet, where nominal attrition ratios are expected to be lower ($a_0 = 0.10$), either the improvement from using penetration aids must be high or the costs and weights must be very low in order for a benefit to be realized.

Design Data for the Jet Flap in Ground Effect

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Introduction

The jet flap is of considerable interest for obtaining very high lifts from an airfoil. This is significant for STOL, for helicopters, and for fluid support of high-speed ground vehicles, or for amplification of conventional flap-type controls. Linearized solutions for the two-dimensional case in an infinite fluid are well-known (for example, Spence[1]). The presence of the ground causes significant effects on a jet flap airfoil, due both to ground-induced pressure and viscous ground effects (blockage). This note distils the linear theory solution for the jet flap in ground effect. The analysis is described in detail in Refs. 2 and 3; here only those results of direct engineering interest are given for use in design.

Theoretical Development

The two-dimensional, inviscid, incompressible steady case is considered (Fig. 1). The problem is to find a potential function, Laplacian in the general field, which satisfies normal boundary conditions on the airfoil, ground plane, and at infinity, and an additional condition on the jet wake, namely no flow through the wake, with the pressure difference across the wake related to the local curvature and the jet blowing coefficient $C_J = J/qc$. Here J is the jet momentum, q the freestream dynamic pressure, and c the chord. Although the field equation is linear, the boundary conditions are not, since on the wake the dynamic condition implies matching terms of order V^2 while the wake shape itself changes under different lifting conditions.

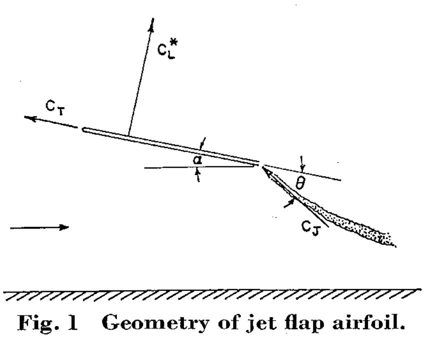

Fig. 1 Geometry of jet flap airfoil.

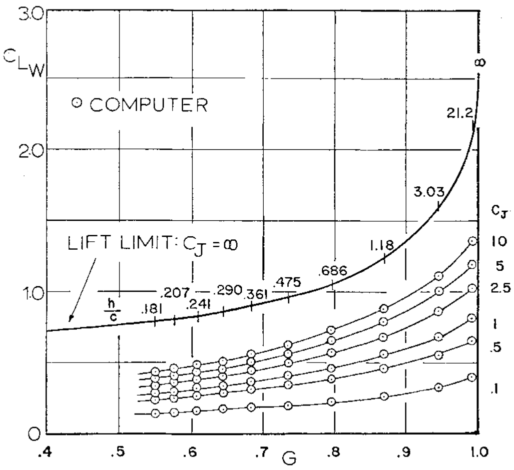

Fig. 2 Wake lift component.

The problem is linearized in the usual thin airfoil fashion, and the boundary conditions applied on some mean line. This still does not explicitly define the boundary condition on the jet wake, but transforms the problem to a linear one with Riemann-Hilbert-Poincare boundary conditions.[3] It may now be considered as four distinct superimposable cases: the classical ones of thickness, camber, and angle of attack (α), and the additional case for variation of the jet efflux angle (θ) (the singular problem). These solutions are functions of C_J and h/c, h being the height of the airfoil.

Details of the technique of solution are not discussed; however, the nose flow is of especial engineering interest. For the case of zero thickness and camber (Fig. 1), where $C_L{}^*$ is the coefficient of pressure lift on the upper and lower surfaces only, and C_T the nose thrust coefficient, $C_L{}^*$ may be eliminated from the two equations of vertical and horizontal equilibrium. The lift coefficient C_L is linear in α and θ. After a further elimination we get

$$C_T = C_J(\theta^2/2) \qquad \alpha = 0 \tag{1}$$

$$C_{L\theta}{}^2 = 2C_JC_{L\alpha} - C_J{}^2 \tag{2}$$

Equation (1) gives a direct insight into the thrust recovery mechanism. For the case $\alpha = 0$, the jet momentum thrust at the trailing edge is $C_J(1 - \theta^2/2 + \ldots)$; it is now clear that the nose suction provides the additional force on the airfoil for full thrust recovery. Physically, the deflection of the jet induces a flow around the nose of a related magnitude. Thus for practical designs attention must be given to the control of the nose flow to realize thrust recovery. Equation (2) shows that it is unnecessary to solve both the angle-of-attack case (for $C_{L\alpha}$) and the singular case (for $C_{L\theta}$) since they are directly related.

Lift of the Airfoil

Having solved the boundary value problem, the pressure distribution may be determined. This is singular, $O(x^{-1/2})$ and $O(C_J \log|c - x|)$, at the leading edge ($x = 0$) and trailing edge ($x = c$). We can isolate these singularities, and express the lift as the sum of three terms. The first, the nose lift C_{LN}, is principally due to pressures near the nose, and appears in closed form as $C_{LN} = 4\pi^{-1/2}C_J{}^{1/2}G^{-1}$. Here G is the geometrical parameter describing the height of the airfoil. For engineering purposes h/c as a function of G may be read off from Fig. 2. Note that $h/c \to 0$, $G \to 0$; $h/c \to \infty$, $G \to 1$. The second component is due directly to the jet momentum, given in linear theory as C_J. The final component (mainly due to pressure near the trailing edge)

Presented as Paper 67-2 at the AIAA 5th Aerospace Sciences Meeting, New York, January 23–26, 1967; submitted January 13, 1967; revision received August 7, 1967. [3.01]
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